

## UNIT-1<sup>ST</sup> - ASSIGNMENT ON DIFFERENTIAL CALCULUS

**Q.1) Use MACLAURIANS THEOREM to solve following ?**

- Find the value of A, B & C in the expansion of the function  $\log(\sec x) = Ax^2 + Bx^4 + Cx^6$
- Expand the function  $f(x) = \log_e(1+x)$
- Find first four terms in expansion of function  $\log(1+\sin x)$  upto the term containing  $x^4$
- Expand the function  $f(x) = \log_e(1+e^x)$
- Expand  $e^{\sin x}$  upto the term containing  $x^4$
- Expand  $e^x \cdot \cos x$
- Expand  $e^x \cdot \sin x$
- Expand  $\tan^{-1}x$  in ascending power of  $x$

**Q.2) Use TAYLOR'S THEOREM to solve following ?**

- Expand  $\sin x$  in power of  $(x - \frac{\pi}{2})$
- Expand  $2x^3 + 7x^2 + x - 1$  in power of  $(x-2)$
- Find the Taylor's series expansion of the function  $f(x) = \log \cos x$  about the point  $\frac{\pi}{3}$
- Expand  $\tan(x + \frac{\pi}{4})$  as far as the term  $x^4$  & evaluate  $\tan 46.5^\circ$  upto four significant fig. digit
- Expand  $\log_e x$  in power of  $(x-1)$  & hence evaluate  $\log_e(1.1)$  correct upto four decimal place
- Compute the approximate value of  $\sqrt{11}$  to four decimal places by taking the first five terms as an approximate Taylor's expansion

**Q.3) Use PARTIAL DIFFERENTIATION method to solve following ?**

- Find all the partial differentiation coefficient of  $f(x,y) = xy + \sin(x+y)$
- If  $x^y + y^x = c$ , Find  $\frac{\partial y}{\partial x}$ .
- If  $u = x \log xy$ , where  $x^3 + y^3 + 3xy = 1$ , find  $\frac{\partial u}{\partial x}$ .
- If  $u = x^2 - y^2 + \sin yz$ , where  $y = e^x$  and  $z = \log_e x$ , find  $\frac{\partial u}{\partial x}$
- If  $u = \tan^{-1} \frac{y}{x} + \sin^{-1} \frac{x}{y}$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$
- If  $u = e^{xyz}$  then show that  $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$

**Q.4) Use EULER'S THEOREM to solve following ?**

a) If  $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$  then show that

$$1) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

$$2) x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = \sin 4u - \sin 2u$$

b) If  $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ , then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$

c) If  $u = \log\left(\frac{x^4+y^4}{x+y}\right)$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$

**Q.5) Use ERROR & APPROXIMATION concept to solve following ?**

a) Evaluate  $\sqrt{99}$  approximately.

b) Evaluate cube root of 127(appx).

c) Find the percentage error in the area of an ellipse if 1% error is made in measuring the major and minor axes.

d) If H.P. is required to propel a steamer is proportional to cube of its velocity and square of its length, prove that 2% increase in velocity and 3% increase in length will require an approximately 12% increase in H.P.

e) The period T of a simple pendulum is given by  $T = 2\pi \sqrt{\frac{l}{g}}$ . Find the maximum error in T due to possible errors upto 1% in l and 2.5% in g.

**Q.6) Use concept of MAXIMA & MINIMA for partial differentiation equation to solve following ?**

a) Discuss the maximum or minimum values of the function  $f(x,y) = x^3 - 4xy + 2y^2$ .

b) Discuss the maxima and minima of the function  $x^3 + y^3 - 3axy$ .

c) Discuss the maximum or minimum value of  $u = x^3 y^2 (1 - x - y)$

d) Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube.

**Q.7) Use concept of TANGENT & NORMAL to solve following ?**

a) Find the equation of tangent at the point (2,2) on the curve  $x^2 - 2xy + y^2 + 2x + y - 6 = 0$

b) Find the equation of tangent at the point  $(a \cos \theta, b \sin \theta)$

**Q.8) Use concept of CENTER OF CURVATURE & RADIUS CURVATURE to solve following ?**

a) Find radius of curvature of  $y = 4 \sin x - \sin 2x$  at  $x = \frac{\pi}{2}$

b) Find the coordinates of the center of curvature for the point (x,y) of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

c) Find the center of curvature at the point (1,-1) of the curve  $y = x^3 - 6x^2 + 3x + 1$ . Hence find the equation of the circle of curvature at this point?