

Agnihotri Engineering & GATE Classes

UNIT- 1st FOURIER SERIES & FOURIER TRANSFORM (Revision Sheet)

Q.1) Obtain the Fourier expansion for $\sqrt{1-\cos x}$ in the interval $-\pi < x < \pi$

Q.2) Prove that $0 \leq x \leq \pi$,

$$x(\pi - x) = \frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \dots \right)$$

Q.3) Find the Fourier series to represent the function $f(x)$, given by

$$f(x) = \begin{cases} x & \text{for } 0 \leq x \leq \pi \\ 2\pi - x & \text{for } \pi \leq x \leq 2\pi \end{cases}$$

$$\text{Deduce that } \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{8}$$

Q.4) Obtain the Fourier expansion

$$f(x) = \begin{cases} -\pi, & \text{if } -\pi < x < 0 \\ x, & \text{if } 0 < x < \pi \end{cases} \text{ and hence deduce that}$$

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

Q.5) Obtain the Fourier series for the function $f(x)$ given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & \text{if } -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & \text{if } 0 \leq x \leq \pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

Q.6) If $f(x) = \begin{cases} 0, & \text{if } -\pi < x < 0 \\ \sin x, & \text{if } 0 < x < \pi \end{cases}$

Prove that

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{\cos 2mx}{4m^2 - 1}. \text{ Hence show that}$$

$$\frac{1}{1.3} - \frac{1}{3.3} + \frac{1}{5.7} - \dots = \frac{1}{4}(\pi - 2).$$

Q.7) Find the Fourier series for the function

$$f(x) = \begin{cases} -K, & -\pi < x < \theta \\ K, & 0 < x < \pi \end{cases}$$

such that $f(x + 2\pi) = f(x)$

and hence show that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$

Q.8) Expand Function $f(x) = x \sin x$ as Fourier series in interval $-\pi \leq x \leq \pi$ and deduce that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \dots = \frac{1}{4}(\pi - 2)$$

Q.9) Obtain the half-range Sine series for $f(x) = 2 - x$ for $0 < x < 2$.

Hence deduce that

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \dots = \frac{\pi}{4}$$

Q.10) Express

$$f(x) = \begin{cases} \frac{1}{4} - x, & \text{if } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{if } \frac{1}{2} < x < 1 \end{cases}$$

Q.11) Find Fourier Transform of following ?

a) $x \exp(-x^2)$

b) $x \exp(-ax^2)$, where $a > 0$.

Classes on **(ED,BEEE,M1,M2,M3,NA,CONTROL,DSP & other GATE oriented Engineering Subjects)**

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