Agnihotri Engineering & GATE Classes

Scripting success stories

UNIT-2ND - ASSIGNMENT ON INTEGRAL CALCULUS

Q.1) Use " Integral as limit of sum concept " to solve following ?

a) $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{2n}\right]$

b) $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\cdots+\frac{1}{8n}\right]$

c) $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+3}+\frac{1}{n+5}+\cdots+\frac{1}{3n-1}\right]$

d) $\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{1}{n}\right)\right]^{1/n}$

e) $\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)^{1/2}\left(1+\frac{3}{n}\right)^{1/3}\ldots\left(1+\frac{1}{n}\right)^{1/n}\right]$

Q.2) Solve following integral using beta & gamma function ?

a) $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}}dx$

b) $\int_0^\infty \sqrt{x}\, e^{-\sqrt[3]{x}}\, dx$

c) $\int_0^2 x\sqrt[3]{(8-x^3)}\, dx$

Prove that-

d) $\int_0^\infty \frac{x^c}{c^x}dx = \frac{\Gamma(c+1)}{\log c^{c+1}}$

e) $\Gamma\left(\frac{3}{2}+x\right)\Gamma\left(\frac{3}{2}-x\right)=\left(\frac{1}{4}-x^2\right)\pi\, sec\pi$

f) State & prove relation between beta & gamma function?

g) State & prove duplication formula?

Q.3) Solve following Double Integration ?

a) $\int_0^1\int_0^x e^{y/x}\, dx\, dy$

b) $\int_0^2\int_0^{\sqrt{1+x^2}}\frac{1}{1+x^2+y^2}dx\, dy$

c) $\int_0^2\int_0^{x^2} e^{y/x}\, dx\, dy$

d) $\int_0^a\int_0^{\sqrt{ay}} xy\, dx\, dy$

Classes on (ED,BEEE,M1,M2,M3,NA,CONTROL,DSP & other GATE oriented Engineering Subjects)

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e) $\int_0^a \int_0^{\sqrt{a^2-y^2}} dy\, dx$

f) $\int_0^1 \int_0^{\sqrt{\frac{1}{2}(1-y^2)}} \frac{1}{\sqrt{1-x^2-y^2}}\, dx\, dy$

Q.4) Solve following triple integration ?

a) $\int_0^3 \int_0^2 \int_0^1 (x+y+z)\,dx\,dy\,dz$

b) $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2+y^2+z^2)\,dz\,dy\,dx$

c) $\int_0^2 \int_0^x \int_0^{x+y} e^x(y+2z)\,dx\,dy\,dz$

d) $\int_0^1 \int_{y^2}^1 \int_0^{1-z} z\,dy\,dz\,dx$

e) $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dx\,dy\,dz$

f) $\int_0^a \int_0^{b\sqrt{\left(1-x^2/a^2\right)}} \int_0^{c\sqrt{\left(1-x^2/a^2-y^2/b^2\right)}} dx\,dy\,dz$

g) $\int_{x=0}^{\log 2} \int_{y=0}^{x} \int_{z=0}^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$

h) $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\,dy\,dx\,dz$

Q.5) Solve following Double Integration by using change of order concept?

a) $\int_0^3 \int_0^{3-x} xy\,dy\,dx$

b) $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$

c) $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\,dx\,dy$

d) $\int_1^2 \int_0^x \frac{1}{x^2+y^2}\,dx\,dy$

e) $\int_0^2 \int_x^{3x-x^2} (3x^2-2xy)\,dx\,dy$

f) $\int_0^3 \int_0^{3-x} x\,dy\,dx$

g) $\int_0^1 \int_{x^2}^{2-x} x\,dy\,dx$