Agnihotri Engineering & GATE Classes

Scripting success stories

UNIT-4TH - ASSIGNMENT ON MATRICES

Q.1) Find rank and nullity of following matrices?

a) $\begin{bmatrix}1 & 2 & 0\\2 & 4 & 7\\1 & 3 & 0\end{bmatrix}$ b) $\begin{bmatrix}1 & 2 & 3\\1 & 3 & 5\\2 & 5 & 8\end{bmatrix}$ c) $\begin{bmatrix}1 & 2 & 3 & 2\\2 & 3 & 5 & 1\\1 & 3 & 4 & 5\end{bmatrix}$ d) $\begin{bmatrix}1 & 3 & 4 & 3\\3 & 9 & 12 & 9\\1 & 3 & 4 & 1\end{bmatrix}$

Q.2) Examine the following equations for consistency & if consistent find the complete solution?

a) x+y+z= -3
3x+y-2z= -2
2x+4y+7z=7

b) x+2y-z= 6
3x-y-2z=3
4x+3y+z=9

c) x+2y-z= 3
3x-y+2z=1
2x-2y+3z=2

d) x+2y+3z=14
3x+y+2z=11
2x+3y+z=11

e) 2x+4y-z=9
3x-y+5z=5
8x+2y+9z=19

f) x-2y+z-w+1=0
3x-2z+3w+4=0
5x-4y+w+3=0

g) 3x-y+z = 0
-15x+6y-5z=0
5x-2y+2z=0

h) x+3y-2z=0
2x-y+4z=0
x-11y+14z=0

i) x-2y+z-w=0
x+y-2z+3w=0
5x-7y+2z-w=0

Q.3) Find Eigen value & Eigen vector of following matrices?

a) $A=\begin{bmatrix}8 & -6 & 2\\-6 & 7 & -4\\2 & -4 & 3\end{bmatrix}$ b) $A=\begin{bmatrix}3 & 1 & 4\\0 & 2 & 6\\0 & 0 & 5\end{bmatrix}$ c) $A=\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}$ d) $A=\begin{bmatrix}-2 & 2 & -3\\2 & 1 & -6\\-1 & -2 & 0\end{bmatrix}$

Q.4) For the following matrices show that the Cayley-Hamilton theorem is satisfied & thus find A^{-1}

a) $A=\begin{bmatrix}2 & -1 & 1\\-1 & 2 & -1\\1 & -1 & 2\end{bmatrix}$ b) $A=\begin{bmatrix}0 & 0 & 1\\3 & 1 & 0\\-2 & 1 & 4\end{bmatrix}$ c) $A=\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}$ d) $A=\begin{bmatrix}1 & 2 & 0\\2 & -1 & 0\\0 & 0 & -1\end{bmatrix}$

Q.5) For what value of λ & μ equations have a) unique solution b) no solution c) ∞ solution ?

a) x+y+z=6
x+2y+3z=10
x+2y+λz=μ

b) x+y+z=1
x+2y+4z=λ
x+4y+10z=λ

ANSWERS

Ans.1 a) ρ(A)=3 N(A)=3 **b)** ρ(A)=3 N(A)=1 **c)** ρ(A)=2 N(A)=ND **d)** ρ(A)=3 N(A)=ND **e)** ρ(A)=3 N(A)=ND **f)** ρ(A)=3 N(A)=ND **g)** ρ(A)=3 N(A)=ND

Ans.2 a) Inconsistent **b)** Unique x=1, y=2, z=-1 **c)** Unique x=-1, y=4, z=4 **d)** Unique x=1, y=2, z=3

e) ∞ solution $x=\frac{-19k+29}{14}$, $y=\frac{13k+17}{14}$, z=k **f)** Inconsistent **g)** Trivial solution x=y=z=0

h) ∞ solution $x=-\frac{10k}{7}$, $y=\frac{8k}{7}$, z=k i) ∞ solution $w=k_1$, $z=k_2$, $x=\frac{3k2-5k1}{3}$, $y=\frac{3k2-4k1}{3}$

Ans.3 a) $x_1=\begin{bmatrix}½\\1\\1\end{bmatrix}$, $x_2=\begin{bmatrix}2\\1\\-2\end{bmatrix}$, $x_3=\begin{bmatrix}2\\-2\\1\end{bmatrix}$ for λ= 0,3,5

b) $x_1=\begin{bmatrix}1\\-1\\0\end{bmatrix}$ $x_2=\begin{bmatrix}1\\0\\0\end{bmatrix}$ $x_3=\begin{bmatrix}3\\2\\1\end{bmatrix}$ for λ=2,3,5

c) $x_1=\begin{bmatrix}1\\2\\0\end{bmatrix}$ $x_2=\begin{bmatrix}-1\\0\\2\end{bmatrix}$ $x_3=\begin{bmatrix}2\\-1\\-1\end{bmatrix}$ for λ= 2,2,8

d) $x_1=\begin{bmatrix}1\\0\\3\end{bmatrix}$ $x_2=\begin{bmatrix}-2\\1\\0\end{bmatrix}$ $x_3=\begin{bmatrix}1\\2\\-1\end{bmatrix}$ for λ=-3,-3,5

Ans.4 a) $\frac{1}{4}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$ b) $\begin{bmatrix}4/5&1/5&-1/5\\-12/5&2/5&3/5\\1&0&0\end{bmatrix}$ *c)* $\begin{bmatrix}-3&0&2\\-1&1/2&1/2\\2&0&-1\end{bmatrix}$ *d)* $\frac{1}{5}\begin{bmatrix}1&2&0\\2&-1&0\\0&0&-5\end{bmatrix}$

Ans.5 a) λ≠3, λ=3 & μ=10, λ=3 & μ≠10

b) at λ=1 :- z=k, x=2k+1, y=-3k

at λ=2 :- x=2k, y=-3k+1, z=k